Perhaps the most interesting and versatile family of relays is the distance-relay group. In the preceding chapter, we examined relays in which one current was balanced against another current, and we saw that the operating characteristic could be expressed as a ratio of the two currents. In distance relays, there is a balance between voltage and current, the ratio of which can be expressed in terms of impedance. Impedance is an electrical measure of distance along a transmission line, which explains the name applied to this group of relays.

THE IMPEDANCE-TYPE DISTANCE RELAY

Since this type of relay involves impedance-type units, let us first become acquainted with them. Generally speaking, the term “impedance” can be applied to resistance alone, reactance alone, or a combination of the two. In protective-relaying terminology, however, an impedance relay has a characteristic that is different from that of a relay responding to any component of impedance. And hence, the term “impedance relay” is very specific.

In an impedance relay, the torque produced by a current element is balanced against the torque of a voltage element. The current element produces positive (pickup) torque, whereas the voltage element produces negative (reset) torque. In other words, an impedance relay is a voltage-restrained overcurrent relay. If we let the control-spring effect be $-K_3$, the torque equation is:

$$T = K_1 I^2 - K_2 V^2 - K_3$$

where $I$ and $V$ are rms magnitudes of the current and voltage, respectively. At the balance point, when the relay is on the verge of operating, the net torque is zero, and

$$K_2 V^2 = K_1 I^2 - K_3$$

Dividing by $K_2 I^2$, we get:

$$\frac{V^2}{I^2} = \frac{K_1}{K_2} \frac{K_3}{K_2 I^2}$$

$$\frac{V}{I} = Z = \sqrt{\frac{K_1}{K_2} - \frac{K_3}{K_2 I^2}}$$
It is customary to neglect the effect of the control spring, since its effect is noticeable only at current magnitudes well below those normally encountered. Consequently, if we let $K_3$ be zero, the preceding equation becomes:

$$Z = \sqrt{\frac{K_1}{K_2}} = \text{constant}$$

In other words, an impedance relay is on the verge of operating at a given constant value of the ratio of $V$ to $I$, which may be expressed as an impedance.

The operating characteristic in terms of voltage and current is shown in Fig. 1, where the effect of the control spring is shown as causing a noticeable bend in the characteristic only at the low-current end. For all practical purposes, the dashed line, which represents a constant value of $Z$, may be considered the operating characteristic. The relay will pick up for any combination of $V$ and $I$ represented by a point above the characteristic in the positive-torque region, or, in other words, for any value of $Z$ less than the constant value represented by the operating characteristic. By adjustment, the slope of the operating characteristic can be changed so that the relay will respond to all values of impedance less than any desired upper limit.

![Operating characteristic of an impedance relay.](image)

A much more useful way of showing the operating characteristic of distance relays is by means of the so-called “impedance diagram” or “$R$-$X$ diagram.” Reference 1 provides a comprehensive treatment of this method of showing relay characteristics. The operating characteristic of the impedance relay, neglecting the control-spring effect, is shown in Fig. 2 on this type of diagram. The numerical value of the ratio of $V$ to $I$ is shown as the
length of a radius vector, such as $Z$, and the phase angle $\theta$ between $V$ and $I$ determines the position of the vector, as shown. If $I$ is in phase with $V$, the vector lies along the $+R$ axis; but, if $I$ is 180 degrees out of phase with $V$, the vector lies along the $-R$ axis. If $I$ lags $V$, the vector has a $+X$ component; and, if $I$ leads $V$, the vector has a $-X$ component. Since the operation of the impedance relay is practically or actually independent of the phase angle between $V$ and $I$, the operating characteristic is a circle with its center at the origin. Any value of $Z$ less than the radius of the circle will result in the production of positive torque, and any value of $Z$ greater than this radius will result in negative torque, regardless of the phase angle between $V$ and $I$.

At very low currents where the operating characteristic of Fig. 1 departs from a straight line because of the control spring, the effect on Fig. 2 is to make the radius of the circle smaller. This does not have any practical significance, however, since the proper application of such relays rarely if ever depends on operation at such low currents.

Although impedance relays with inherent time delay are encountered occasionally, we shall consider only the high-speed type. The operating-time characteristic of a high-speed impedance relay is shown qualitatively in Fig. 3. The curve shown is for a particular value of current magnitude. Curves for higher currents will lie under this curve, and curves for

Fig. 2. Operating characteristic of an impedance relay on an R-X diagram.
lower currents will lie above it. In general, however, the operating times for the currents usually encountered in normal applications of distance relays are so short as to be within the definition of high speed, and the variations with current are neglected. In fact, even the increase in time as the impedance approaches the pickup value is often neglected, and the time curve is shown simply as in Fig. 4.

Various types of actuating structure are used in the construction of impedance relays. Inverse-time relays use the shaded-pole or the watt-metric structures. High-speed relays may use a balance-beam magnetic-attraction structure or an induction-cup or double-loop structure.

For transmission-line protection, a single-phase distance relay of the impedance type consists of a single-phase directional unit, three high-speed impedance-relay units, and a timing unit, together with the usual targets, seal-in unit, and other auxiliaries. Figure 5 shows very schematically the contact circuits of the principal units. The three impedance units are labeled $Z_1$, $Z_2$, and $Z_3$. The operating characteristics of these three units are independently adjustable. On the $R$-$X$ diagram of Fig. 6, the circle for $Z_1$ is the smallest, the circle for $Z_3$ is the largest, and the circle for $Z_2$ is intermediate. It will be evident, then, that
any value of impedance that is within the $Z_1$ circle will cause all three impedance units to operate. The operation of $Z_1$ and the directional unit will trip a breaker directly in a very short time, which we shall call $T_1$. Whenever $Z_3$ and the directional unit operate, the timing unit is energized. After a definite delay, the timing unit will first close its $T_2$ contact, and later its $T_3$ contact, both time delays being independently adjustable. Therefore, it can be seen that a value of impedance within the $Z_2$ circle, but outside the $Z_1$ circle, will result in tripping in $T_2$ time. And finally, a value of $Z$ outside the $Z_1$, and $Z_2$ circles, but within the $Z_3$ circle, will result in tripping in $T_3$ time.

It will be noted that, if tripping is somehow blocked, the relay will make as many attempts to trip as there are characteristic circles around a given impedance point. However, use may not be made of this possible feature.

Figure 6 shows also the relation of the directional-unit operating characteristic to the impedance-unit characteristics on the same $R$-$X$ diagram. Since the directional unit permits tripping only in its positive-torque region, the inactive portions of the impedance-unit characteristics are shown dashed. The net result is that tripping will occur only for points that are both within the circles and above the directional-unit characteristic.

Because this is the first time that a simple directional-unit characteristic has been shown on the $R$-$X$ diagram, it needs some explanation. Strictly speaking, the directional unit has a straight-line operating characteristic, as shown, only if the effect of the control spring is neglected, which is to assume that there is no restraining torque. It will be recalled that, if we neglect the control-spring effect, the torque of the directional unit is:

$$T = K_1 VI \cos (\theta - \tau)$$
When the net torque is zero,

\[ K_1 VI \cos (\theta - \tau) = 0 \]

Since \( K_1, V, \) or \( I \) are not necessarily zero, then, in order to satisfy this equation,

\[ \cos (\theta - \tau) = 0 \]

or

\[ (\theta - \tau) = \pm 90^\circ \]

Hence, \( \theta = \tau = \pm 90^\circ \) describes the characteristic of the relay. In other words, the head of any radius vector \( Z \) at \( 90^\circ \) from the angle of maximum torque lies on the operating characteristic, and this describes the straight line shown on Fig. 6, the particular value of \( \tau \) having been chosen for reasons that will become evident later.

We should also develop the operating characteristic of a directional relay when the control-spring effect is taken into account. The torque equation as previously given is:

\[ T = K_1 VI \cos (\theta - \tau) - K_2 \]

At the balance point, the net torque is zero, and hence:

\[ K_1 VI \cos (\theta - \tau) = K_2 \]
But \( I = \frac{V}{Z} \), and hence:

\[
\frac{V^2}{Z} \cos (\theta - \tau) = \frac{K_2}{K_1}
\]

or

\[
Z = \frac{K_1}{K_2} V^2 \cos (\theta - \tau)
\]

This equation describes an infinite number of circles, one for each value of \( V \), one circle of which is shown in Fig. 7 for the same relay connections and the same value of \( \tau \) as in Fig. 6. The fact that some values of \( \theta \) will give negative values of \( Z \) should be ignored. Negative \( Z \) has no significance and cannot be shown on the \( R-X \) diagram.

![Fig. 7. The characteristics of a directional relay for one value of voltage.](image)

The centers of all the circles will lie on the dashed line directed from \( O \) through \( M \), which is at the angle of maximum torque. The diameter of each circle will be proportional to the square of the voltage. At normal voltage, and even at considerably reduced voltages, the diameter will be so large that for all practical purposes we may assume the straight-line characteristic of Fig. 6.
Looking somewhat ahead to the application of distance relays for transmission-line protection, we can show the operating-time-versus-impedance characteristic as in Fig. 8. This characteristic is generally called a “stepped” time-impedance characteristic. It will be shown later that the $Z_1$ and $Z_2$ units provide the primary protection for a given transmission-line section, whereas $Z_2$ and $Z_3$ provide back-up protection for adjoining busses and line sections.

**THE MODIFIED IMPEDANCE-TYPE DISTANCE RELAY**

The modified impedance-type distance relay is like the impedance type except that the impedance-unit operating characteristics are shifted, as in Fig. 9. This shift is accomplished by what is called a “current bias,” which merely consists of introducing into the voltage supply an additional voltage proportional to the current,² making the torque equation as follows:

$$T = K_1 I^2 - K_2 (V + CI)^2$$

The term $(V + CI)$ is the rms magnitude of the vector addition of $V$ and $CI$, involving the angle $\theta$ between $V$ and $I$ as well as a constant angle in the constant $C$ term. This is the equation of a circle whose center is offset from the origin, as shown in Fig. 9. By such biasing, a characteristic circle can be shifted in any direction from the origin, and by any desired amount, even to the extent that the origin is outside the circle. Slight variations may occur in the biasing, owing to saturation of the circuit elements. For this reason, it is
not the practice to try to make the circles go through the origin, and therefore a separate
directional unit is required as indicated in Fig. 9.

Since this relay is otherwise like the impedance-type relay already described, no further
description will be given here.

THE REACTANCE-TYPE DISTANCE RELAY

The reactance-relay unit of a reactance-type distance relay has, in effect, an overcurrent
element developing positive torque, and a current-voltage directional element that either
opposes or aids the overcurrent element, depending on the phase angle between the
current and the voltage. In other words, a reactance relay is an overcurrent relay with
directional restraint. The directional element is arranged to develop maximum negative
torque when its current lags its voltage by 90°. The induction-cup or double-induction-loop
structures are best suited for actuating high-speed relays of this type.

If we let the control-spring effect be $-K_3$, the torque equation is:

$$ T = K_1 I^2 - K_2 V I \sin \theta - K_3 $$

where $\theta$ is defined as positive when $I$ lags $V$. At the balance point, the net torque is zero,
and hence;

$$ K_1 I^2 = K_2 V I \sin \theta + K_3 $$
Dividing both sides of the equation by \( I_2 \), we get:

\[
K_1 = K_2 \frac{V}{I} \sin \theta + \frac{K_3}{I^2}
\]

or

\[
\frac{V}{I} \sin \theta = Z \sin \theta = X = \frac{K_1}{K_2} - \frac{K_3}{K_2 I^2}
\]

If we neglect the effect of the control spring,

\[
X = \frac{K_1}{K_2} = \text{constant}
\]

In other words, this relay has an operating characteristic such that all impedance radius vectors whose heads lie on this characteristic have a constant \( X \) component. This describes the straight line of Fig. 10. The significant thing about this characteristic is that the resistance component of the impedance has no effect on the operation of the relay; the relay responds solely to the reactance component. Any point below the operating characteristic—whether above or below the \( R \) axis—will lie in the positive-torque region.

Taking into account the effect of the control spring would lower the operating characteristic toward the \( R \) axis and beyond at very low values of current. This effect can be neglected in the normal application of reactance relays.

It should be noted in passing that, if the torque equation is of the general form

\[
T = K_1 I^2 - K_2 VI \cos (\theta - \tau) - K_3,
\]

and if \( \tau \) is made some value other than 90°, a straight-line operating characteristic will still be obtained, but it will not be parallel to the \( R \) axis. This general form of relay has been called an “angle-impedance” relay.

A reactance-type distance relay for transmission-line protection could not use a simple directional unit as in the impedance-type relay, because the reactance relay would trip under normal load conditions at or near unity power factor, as will be seen later when we consider what different system-operating conditions “look” like on the \( R-X \) diagram. The reactance-type distance relay requires a directional unit that is inoperative under normal load conditions. The type of unit used for this purpose has a voltage-restraining element
that opposes a directional element, and it is called an “admittance” or “mho” unit or relay. In other words, this is a voltage-restrained directional relay. When used with a reactance-type distance relay, this unit has also been called a “starting unit.” If we let the control-spring effect be $-K_3$, the torque of such a unit is:

$$T = K_1 VI \cos(\theta - \tau) - K_2 V^2 - K_3$$

where $\theta$ and $\tau$ are defined as positive when $I$ lags $V$. At the balance point, the net torque is zero, and hence:

$$K_2 V^2 = K_1 VI \cos(\theta - \tau) - K_3$$

Dividing both sides by $K_2 VI$, we get:

$$\frac{V}{I} = Z = \frac{K_1}{K_2} \cos(\theta - \tau) - \frac{K_3}{K_2 VI}$$

If we neglect the control-spring effect,

$$Z = \frac{K_1}{K_2} \cos(\theta - \tau)$$

It will be noted that this equation is like that of the directional relay when the control-spring effect is included, but that here there is no voltage term, and hence the relay has but one circular characteristic.

The operating characteristic described by this equation is shown in Fig. 11. The diameter of this circle is practically independent of voltage or current, except at very low magnitudes of current or voltage when the control-spring effect is taken into account, which causes the diameter to decrease.

![Diagram of operating characteristic of a directional relay with voltage restraint.](image)
The complete reactance-type distance relay has operating characteristics as shown in Fig. 12. These characteristics are obtained by arranging the various units as described in Fig. 5 for the impedance-type distance relay. It will be observed here, however, that the directional or starting unit (S) serves double duty, since it not only provides the directional function but also provides the third step of distance measurement with inherent directional discrimination.

The time-versus-impedance characteristic is the same as that of Fig. 8.
THE MHO-TYPE DISTANCE RELAY

The mho unit has already been described, and its operating characteristic was derived in connection with the description of the starting unit of the reactance-type distance relay. The induction-cylinder or double-induction-loop structures are used in this type of relay.

The complete distance relay for transmission-line protection is composed of three high-speed mho units \((M_1, M_2, \text{ and } M_3)\) and a timing unit, connected in a manner similar to that shown for an impedance-type distance relay, except that no separate directional unit is required, since the mho units are inherently directional.\(^3\) The operating characteristic of the entire relay is shown on Fig. 13.

![Fig. 13. Operating characteristics of a mho-type distance relay.](image)

The operating-time-versus-impedance characteristic of the mho-type distance relay is the same as that of the impedance-type distance relay, Fig. 8.

By means of current biasing similar to that described for the offset impedance relay, a mho-relay characteristic circle can be offset so that either it encircles the origin of the \(R\)-\(X\) diagram or the origin is outside the circle.
GENERAL CONSIDERATIONS APPLICABLE TO ALL DISTANCE RELAYS

OVERREACH

When a short circuit occurs, the current wave is apt to be offset initially. Under such conditions, distance relays tend to “overreach,” i.e., to operate for a larger value of impedance than that for which they are adjusted to operate under steady-state conditions. This tendency is greater, the more inductive the impedance is. Also, the tendency is greater in electromagnetic-attraction-type relays than in induction-type relays. The tendency to overreach is minimized in the design of the relay-circuit elements, but it is still necessary to compensate for some tendency to overreach in the adjustment of the relays. Compensation for overreach as well as for inaccuracies in the current and voltage sources is obtained by adjusting the relays to operate at 10% to 20% lower impedance than that for which they would otherwise be adjusted. This will be further discussed when we consider the application of these relays.

MEMORY ACTION

Relays in which voltage is required to develop pickup torque, such as mho-type relays or directional units of other relays, may be provided with “memory action.” Memory action is a feature that can be obtained by design in which the current flow in a voltage-polarizing coil does not cease immediately when the voltage on the high-voltage side of the supply-voltage transformer is instantly reduced to zero. Instead, the stored energy in the voltage circuit causes sinusoidal current to flow in the voltage coil for a short time. The frequency of this current and its phase angle are for all practical purposes the same as before the high-tension voltage dropped to zero, and therefore the relay is properly polarized since, in effect, it “remembers” the voltage that had been impressed on it. It will be evident that memory action is usable only with high-speed relays that are capable of operating within the short time that the transient polarizing current flows. It will also be evident that a relay must have voltage applied to it initially for memory action to be effective; in other words, memory action is ineffective if a distance relay’s voltage is obtained from the line side of a line circuit breaker and the breaker is closed when there is a short circuit on the line.

Actually, it is a most rare circumstance when a short circuit reduces the relay supply voltage to zero. The short circuit must be exactly at the high-voltage terminals of the voltage transformer, and there must be no arcing in the short circuit. About the only time that this can happen in practice is when maintenance men have forgotten to remove protective grounding devices before the line breaker is closed. The voltage across an arcing short circuit is seldom less than about 4% of normal voltage, and this is sufficient to assure correct distance-relay operation even without the help of memory action.

Memory action does not adversely affect the distance-measuring ability of a distance relay. Such ability is important only for impedance values near the point for which the operating time steps from $T_1$ to $T_2$ or from $T_2$ to $T_3$. For such impedances, the primary voltage at the relay location does not go to zero, and the effect of the transient is “swamped.”
THE VERSATILITY OF DISTANCE RELAYS

It is probably evident from the foregoing that on the R-X diagram we can construct any desired distance-relay operating characteristic composed of straight lines or circles. The characteristics shown here have been those of distance relays for transmission-line protection. But, by using these same characteristics or modifications of them, we can encompass any desired area on the R-X diagram, or we can divide the diagram into various areas, such that relay operation can be obtained only for certain relations between $V$, $I$, and $\theta$. That this is a most powerful tool will be seen later when we learn what various types of abnormal system conditions “look” like on the $R$-$X$ diagram.

THE SIGNIFICANCE OF Z

Since we are accustomed to associating impedance with some element such as a coil or a circuit of some sort, one might well ask what the significance is of the impedance expressed by the ratio of the voltage to the current supplied to a distance relay. To answer this question completely at this time would involve getting too far ahead of the story. It depends, among other things, on how the voltage and current supplied to the relay are obtained. For the protection of transmission lines against short circuits, which is the largest field of application of distance relays, this impedance is proportional, within certain limits, to the physical distance from the relay to the short circuit. However, the relay will still be energized by voltage and current under other than short-circuit conditions, such as when a system is carrying normal load, or when one part of a system loses synchronism with another, etc. Under any such condition, the impedance has a different significance from that during a short circuit. This is a most fascinating part of the story, but it must wait until we consider the application of distance relays.

At this point, one may wonder why there are different types of distance relays for transmission-line protection such as those described. The answer to this question is largely that each type has its particular field of application wherein it is generally more suitable than any other type. This will be discussed when we examine the application of these relays. These fields of application overlap more or less, and, in the overlap areas, which relay is chosen is a matter of personal preference for certain features of one particular type over another.

PROBLEMS

1. On an $R$-$X$ diagram, show the impedance radius vector of a line section having an impedance of $2.8 + j5.0$ ohms. On the same diagram, show the operating characteristics of an impedance relay, a reactance relay, and a mho relay, each of which is adjusted to just operate for a zero-impedance short circuit at the end of the line section. Assume that the center of the mho relay’s operating characteristic lies on the line-impedance vector.

Assuming that an arcing short circuit having an impedance of $1.5 + j\theta$ ohms can occur anywhere along the line section, show and state numerically for each type of relay the maximum portion of the line section that can be protected.

2. Derive and show the operating characteristic of an overcurrent relay on an $R$-$X$ diagram.
3. A current-voltage directional relay has maximum torque when the current leads the voltage by 90°. The voltage coil is energized through a voltage regulator that maintains at the relay terminals a voltage that is always in phase with—and of the same frequency as—the system voltage, and that is constant in magnitude regardless of changes in the system voltage.

Derive the equation for the relay’s operating characteristic in terms of the system voltage and current, and show this characteristic on an $R-X$ diagram.

4. Write the torque equation, and derive the operating characteristic of a resistance relay.

**BIBLIOGRAPHY**


